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A Renormalization Prescription for Massless Quantum Electrodynamics

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Abstract. A new perturbative renormalization scheme is introduced for massless QED_4. The method rests on the renormalized tree expansion rather than dimensional renormalization, and so does not rely on any detailed analyticity arguments. The infrared convergence of the gauge invariant prescription is seen to be a consequence of the Ward identity satisfied by the theory.

1. Introduction

Perturbative QED_4 with a massless electron requires a renormalization scheme which eliminates ultraviolet (UV) and infrared (IR) divergences while preserving a Ward identity such as that satisfied formally by the unrenormalized theory

$$Z = \int dA d\tilde{\psi} d\psi \exp - \int [\tfrac{1}{4}F^2 - \tfrac{1}{2}(\partial \cdot A)^2 + \tilde{\psi}(-i\partial\!\!\!/ + m + |e|A\!\!\!/)\psi] . \tag{1.1}$$

The general method of dimensional renormalization pioneered by 't Hooft and Veltman ['tHV] and refined by Becchi, Rouet, and Stora [BRS] and Breitenlohner and Maison [BM] apparently provides us with just such a scheme. Here, we present a direct alternative proof of the perturbative gauge invariance and renormalizability of massless QED_4 (using the Pauli-Villars loop/photon regularization). It is based on the tree expansion renormalization method introduced by Gallavotti and Nicolo [GN], as developed and applied to massive QED_4 by Feldman, Hurd, Rosen, and Wright [FHRW]. The proof of the $m \to 0$ limit, not considered in [FHRW], follows the lines anticipated in [BM]: "A proof of the Ward identities expressing the symmetry at the level of Green's (or vertex) functions can be given to all orders of perturbation theory inductively hand in hand with a proof of the absence of IR-counterterms in the Lagrangian."

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