Commun. Math. Phys. 120, 409–436 (1989)

Communications in Mathematical Physics

Asymptotics of the Susceptibility for the Ising Model on the Hierarchical Lattices

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Abstract. Using the GKS inequality Griffiths and Kaufman showed in [12] that the susceptibility in the Ising model on the hierarchical lattices is infinite for $T > T_c$. In the present work we give an analytic calculation of the dominant singularity of the susceptibility when $h \to 0$ for $T > T_c$. Moreover we analytically find the spontaneous magnetization for $T < T_c$ and prove that for $T < T_c$ the susceptibility is finite.

1. Introduction

In the present paper the Ising model on the "diamond hierarchical lattices" (DHL) is considered. The models of such type were introduced first by Migdal [1] and Kadanoff [2] in the context of an approximate real space renormalization group (RG) scheme for the Ising model on Z^d. In the following papers of Berker and Ostlund [3] and Bleher and Žalys [4] it was shown that the Migdal-Kadanoff RG equations become exact for the models on DHL. In the fundamental paper of Griffiths and Kaufman [5] general definitions, examples and main properties of the hierarchical lattices (HL) were given and discussed (the term "HL" was introduced first in [5]). After that in many papers (see [6–13]) various spin models on the HL were studied as independent (from any approximate RG scheme) objects of the investigation. In our previous paper [14] we analysed (following [5]) the thermodynamical limit of the ferromagnetic Ising model on the HL and proved that the limit Gibbs state is unique unless $T < T_c$, $h = 0$, and in that case the number of pure Gibbs states is equal to 2 (+ and − phases).

An intriguing result was obtained in [12] where it was shown that in the Ising model on the HL the susceptibility is infinite for $T > T_c$. The main idea of the paper [12] was a calculation of the (dimensionless) susceptibility at $T = \infty$ (non-interacting spins) which turned out to be infinite. Then the required result was obtained using the GKS inequality which states the magnetization increases with decreasing of T.

Here, we calculate analytically the dominant singularity of the susceptibility when $h \to 0$ for $T > T_c$. It turns out to be logarithmic in the two-dimensional case