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Kac-Moody Monopoles and Periodic Instantons*

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Abstract. It is shown that calorons, or periodic instantons are the same as monopoles with the loop group as their structure group. Their twistor correspondences and spectral data are defined. The spectral data is shown to determine the general caloron.

Introduction

The motivation for this work was the observation, explained in Sect. 1, that the self-duality equations for a periodic instanton could be re-interpreted as the Bogomolny equations for a monopole whose structure group is the loop group if the degree operator ia adjoined to the loop algebra in the usual way. We shall follow the example of Nahm 1983 and call these objects calorons. Except for Sect. 7 we shall concentrate on periodic instantons for $SU(n)$.

Because a caloron has these two interpretations there are two twistor correspondences that can be applied to it. This was first considered for $SU(2)$ by Hitchin in an unpublished work. In Sect. 2 we follow the idea of Hitchin and apply the twistor correspondence for instantons as in Atiyah, Hitchin, and Singer (1978). This shows that the caloron is equivalent to a holomorphic bundle on $\hat{\mathscr{T}}$ the twistor space of $S^1 \times R^3$. If instead the caloron is regarded as a loop group monopole then the twistor correspondence of Hitchin (1982) and Murray (1984) can be applied to show that it is equivalent to an infinite rank holomorphic bundle on the minitwistor space TP_1. The twistor space $\hat{\mathscr{T}}$ is a $C^\times$ fibering over TP_1 and the two holomorphic bundles are related by pushing down these fibres.

A monopole for a compact group has associated to it a collection of algebraic curves indexed by the Dynkin diagram of the group (see Murray, 1984). When two nodes are joined on the Dynkin diagram the intersection of the corresponding curves is split into two pieces. These curves and the splitting are an invariant of the monopole called the spectral data. When the intersections of the curves are finite the monopole is called general and is determined by the spectral data, see Hitchin (1982), Murray (1984), and Hurtubise and Murray (1988b). These spectral curves

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