Commun. Math. Phys. 120, 269–294 (1988)

Communications in Mathematical Physics

The Connection Whose Holonomy is the Classical Adiabatic Angles of Hannay and Berry and Its Generalization to the Non-Integrable Case

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Abstract. We show how averaging defines an Ehresmann connection whose holonomy is the classical adiabatic angles which Hannay defined for families of completely integrable systems. The averaging formula we obtain for the connection only requires that the family of Hamiltonians has a continuous symmetry group. This allows us to extend the notion of the Hannay angles to families of non-integrable systems with symmetry. We state three geometric axioms satisfied by the connection. These axioms uniquely determine the connection, thus enabling us to find new formulas for the connection and its curvature. Two examples are given.

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Introduction

In (1984) Berry discovered a phase associated to a family of self-adjoint operators. This phase is determined by a loop in the space which parametrizes the family, and by initial choice of nondegenerate eigenvalue. In (1985) Hannay found a classical analogue of this phase. This analogue, called the classical adiabatic angles, or Hannay angles, is associated to a family of completely integrable systems. The