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Quantum System in Contact with a Thermal Environment: Rigorous Treatment of a Simple Model*

P. de Smedt[1,2], D. Dürr[1,3], J. L. Lebowitz[1,4] and C. Liverani[1]

[1] Department of Mathematics, Rutgers University, New Brunswick, NJ 08903, USA
[2] Senior Research Assistant of the Belgian National Fund for Scientific Research: New permanent address: Koninklijke Laboratoria Shell, Amsterdam, The Netherlands
[3] Heisenberg fellow, Permanent address: Bibos Universität Bielefeld, D-4800 Bielefeld, FRG
[4] Department of Physics, Rutgers University, New Brunswick, NJ 08903, USA

Abstract. We study the quantum dynamics of a particle of mass M in an external potential $V(Q)$, interacting with a simple model environment—a harmonic chain of $2N$ particles with mass m and spring constant k. The classical version of this model was studied by Rubin and is equivalent to standard models of a particle interacting with a phonon bath. Setting $m = m^*/L$ and $k = k^*L$, we prove that for a suitable class of potentials V and initial states ω_0, the time evolution of the mass M particle converges, when $N \to \infty$ and $L \to \infty$, to the time evolution governed by the Quantum Langevin Equation (QLE) which has been found by Ford, Kac and Mazur. Furthermore we show that, for this class of potentials, the QLE has a unique solution for all positive times, such solution can be expressed as a convergent expansion in the deviation of $V(Q)$ from a harmonic potential. The equilibrium properties of the particle with mass M can be expressed in terms of an integral, over path space, with a Gaussian measure which has mean zero and covariance proportional to $[-\Delta + \eta h/M\sqrt{-\Delta}]^{-1}$; where $\eta = 2\sqrt{km}$ is the friction constant, and h is the Plancks' constant (divided by 2π).

1. Introduction

The behaviour of quantum systems in a dissipative environment is a problem of fundamental and continuing interest [1]. More precisely, one wishes to study the quantum dynamics of a small system (or selected degrees of freedom) in contact with a much larger system—the latter is to be thought of as a heat bath with an essentially infinite number of degrees of freedom. These dynamics are to take account, in a hopefully simple way, of the esssential effects produced by the environment. We shall use the letters S, E, and T to denote respectively the system, the environment and their union, i.e. the total (isolated) system.

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