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Symanzik's Improved Actions from the Viewpoint of the Renormalization Group

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Abstract. We investigate Symanzik's improvement program in a four-dimensional Euclidean scalar field theory with smooth momentum space cutoff. We use Wilson's renormalization group transformation to define the improved actions as a sequence of initial data for the effective action at the fundamental cutoff. This leads to a sequence of solutions to the renormalization group equation. We define the parameters of the improved actions implicitly by conditions on the effective action at a renormalization scale. The improved actions are close approximations to the continuum effective action. We prove their existence to every order of improvement and to every order of renormalized perturbation theory.

1. Introduction

Wilson's effective actions [Wil 74] have become an important tool in quantum field theory. They have proved to be useful in establishing perturbative renormalizability without relying on detailed estimates on Feynman diagrams [Pol 84]. Work in constructive field theory also demonstrates that the investigation of effective actions provides a natural tool to understand the full theory, at least in the case of asymptotically free models [GK 84].

In this paper we address the question of the locality properties of effective actions. Generally effective actions are infinite sums of nonlocal terms. The question arises how to find good, i.e. local, approximations. This important problem has received little attention except within Monte Carlo renormalization group studies [Gup 85].

Symanzik's improved actions [Sym 83] are candidates for local approximations to effective actions. They were found by Symanzik in a different context and he did not investigate their relationship with the renormalization group. The parameters of Symanzik's improved actions are determined by demanding a

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