

Periodic Solutions of Some Infinite-Dimensional Hamiltonian Systems Associated with Non-Linear Partial Difference Equations. II

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Abstract. This is our second paper devoted to the study of some non-linear Schrödinger equations with random potential. We study the non-linear eigenvalue problems corresponding to these equations. We exhibit a countable family of eigenfunctions corresponding to simple eigenvalues densely embedded in the “band tails.” Contrary to our results in the first paper, the results established in the present paper hold for an arbitrary strength of the non-linear (cubic) term in the non-linear Schrödinger equation.

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5. Introduction

In this paper we continue our analysis of the non-linear eigenvalue problem

$$\left. \begin{aligned} &(-\Delta + V(x) + \lambda W(u(x))u(x) = Eu(x)) \\ &u \in \ell^2(\mathbb{Z}^v), \quad \|u\|_2 = 1, \end{aligned} \right\} \quad (5.1)$$

which was initiated in [1]. In (5.1), Δ is the finite difference Laplacian,

$$(\Delta u)(x) = \sum_{y: |y-x|=1} u(y), \quad (5.2)$$

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