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Large Momentum Properties and Wilson Short Distance Expansion in Non-Perturbative Field Theory

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Abstract. Large momentum properties and Wilson-Zimmerman short-distance expansion are established via phase-space analysis for the weakly coupled massive Gross-Neveu model in dimension 2. Methods are applicable more generally.

1. Introduction

The aim of this work is to show how large momentum and (related) short-distance properties can be rigorously established in non-perturbative field theory via a refined application of phase-space analysis. We consider for definiteness the massive, weakly coupled Gross-Neveu model in dimension 2 [1] whose renormalization parts are as in φ_4^4 2- and 4-point functions, and which exists as a non-trivial, asymptotically free theory recently constructed at weak (renormalized) coupling in [2, 3]. Methods are, however, applicable more generally.

Results in Sects. 2 and 3 apply to euclidean functions. The extension to cases when some of the variables are non-euclidean (and in fact are fixed in Minkowski energy-momentum space while others tend to infinity in euclidean space) is discussed in Sect. 4: as recalled below, this is needed e.g. for "field theory" versions of Wilson-Zimmerman short-distance expansion.

Large momentum properties of the 4-point connected, amputated function $F(p_1, \ldots, p_4)$ are established in Sect. 2. The method is based on a suitable modification of the renormalization procedure of [2, 4]: effective couplings of vertices involving external lines will depend on corresponding energy-momenta. The reason for this modification is explained in Sect. 2.1. It amounts to the introduction of effective couplings for composite operators.

Results include in particular

(i) "Generic" situations

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