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The $A_n^{(1)}$ Face Models

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Abstract. Presented here is the construction of solvable two-dimensional lattice models associated with the affine Lie algebra $A_n^{(1)}$ and an arbitrary pair of Young diagrams. The models comprise two kinds of fluctuation variables; one lives on the sites and takes on dominant integral weights of a fixed level, the other lives on edges and assumes the weights of the representations of $sl(n+1,\mathbf{C})$ specified by Young diagrams. The Boltzmann weights are elliptic solutions of the Yang–Baxter equation. Some conjectures on the one point functions are put forth.

1. Introduction

Let us begin with a link between solvable lattice models and the representation theory of $A_1^{(1)}$. It has been found in [1–4] through the computation of the one point functions $P(a)$ of the former; e.g., in Regime III of the models treated in [2–4], they were completely determined by the decomposition of characters

$$\chi_\xi \chi_\eta = \sum_a b_{\xi\eta a} \chi_a$$

for the pair $A_1^{(1)} \oplus A_1^{(1)} \supset A_1^{(1)}$ and the branching coefficients $b_{\xi\eta a}$ appearing therein:

$$P(a) = \frac{b_{\xi\eta a} \chi_a}{\chi_\xi \chi_\eta}.$$

In this paper we take one more step towards a thorough understanding of this phenomenon.

The corner transfer matrix method—the trick in the computation—was originally invented by Baxter [5]. In his study of the hard hexagon model there emerged a remarkable role of the q-series identities of the Rogers–Ramanujan type. A series of models with interactions round a face were then worked out [6] and their critical exponents were identified with those of the minimal conformal field theories [7]. Further studies along this line were pursued by several authors [8–11] until the complete result was obtained in [3, 4]. There appeared the affine