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The Extended Phase Space of the BRS Approach

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Abstract. The origin of the classical BRS symmetry is discussed for the case of a first class constrained system consisting of a $2n$-dimensional phase space S with free action of a Lie gauge group G of dimension m. The extended phase space S_{ext} of the Fradkin-Vilkovisky approach is identified with a globally trivial vector bundle over S with fibre $L^*(G) \oplus L(G)$, where $L(G)$ is the Lie algebra of G and $L^*(G)$ its dual. It is shown that the structure group of the frame bundle of the supermanifold S_{ext} is the orthosymplectic group $\mathrm{OSp}(m,m;2n)$, which is the natural invariance group of the super Poisson bracket structure on the function space $C^\infty(S_{\text{ext}})$. The action of the BRS operator Ω is analyzed for the case $S=R^{2n}$ with constraints given by pure momenta. The breaking of the $\mathrm{osp}(m,m;2n)$-invariance down to $\mathrm{sp}(2n-2m)$ occurs via an intermediate "$\mathrm{osp}(m;2n-m)$." Starting from a $(2n+2m)$-dimensional system with orthosymplectic invariance, different choices for the BRS operator correspond to choosing different $2n$-dimensional constraint supermanifolds in S_{ext}, which in general characterize different constrained systems. There is a whole family of physically equivalent BRS operators which can be used to describe a particular constrained system.

I. Introduction. Use of BRS Methods and Ghosts

A prominent feature in the use of BRS methods in field theory is the appearance of so-called ghosts. Ghosts were first introduced in the context of a path integral approach to gauge field theory. According to Faddeev and Popov [FAD/POP] the functional measure (after gauge fixing) has to contain a determinantal factor which accounts for the fact that we have to factor out by the volume of the gauge group. This way we get rid of the redundancy in the dynamics which is due to the presence of unphysical gauge degrees of freedom.

Ghost fields appear during the computation of Feynman diagrams when one rewrites this determinant in exponential form (using Berezin integration for anticommuting Grassmann fields) to arrive at an effective action. As was first noticed by Becchi, Rouet, and Stora [B/R/S], this effective action possesses a new