Commun. Math. Phys. 119, 501–508 (1988)

Communications in Mathematical Physics


# Convergence in the Linked Cluster Theorem for Many Body Fermion Systems

V. A. Malyshev

Department of Mathematics and Mechanics, Moscow State University, SU-117234 Moscow, USSR

**Abstract.** We consider small coupling fermion systems with ultraviolet and space cutoffs and prove that they are asymptotically complete. The main ingredient is the convergence in the real time linked cluster theorem.

Here we prove some conjectures which are either explicit or implicit in the classical books by Friedrichs [1] and Hepp [2].

One of the main tools in the quantum many-body calculations is the linked cluster theorem (see Theorems 2.7 and 2.8 in [2]), which up to now was a purely formal result even with the adiabatic cutoff. Here the convergence of the corresponding series is proven. The corollary is asymptotic completeness. We stress that convergence is given in real time, using some experience in the euclidean region. In general, cluster expansions in real time (necessary for asymptotic completeness) seem to be more involved than in the euclidean region.

## 1. Notations

We use mostly notations and definitions of [2]. Let us consider the antisymmetric Fock space $\mathscr{F}=\mathscr{F}(L_2(R^\nu))$ over $L_2(R^\nu)$; further on $\nu\geqq 3$, $a^*(f)$ and $a(f)$ being creation and annihilation operators, satisfying

$$a^*(f)a(g)+a(g)a^*(f)=(f,g), \quad f,g\in L_2(R^\nu), \tag{1}$$

where the scalar product $(f,g)$ in $L_2(R^\nu)$ is antilinear in the second argument. Let us denote by $\mathscr{F}_0(S)$ the set of vectors

$$(f_0,f_1,\ldots,f_n,0,0,\ldots)$$

in $\mathscr{F}$ such that $f_n(x_1,\ldots,x_n)\in S(R^{\nu n})$.

We consider hamiltonains

$$H=H_0+\lambda V$$