Quasi-Genericity of Bifurcations to High Dimensional Invariant Tori for Maps

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Abstract. We consider a family of maps in a Banach space E near the situation when the derivative at the fixed point has two pairs of complex eigenvalues lying on the unit circle, the other part of the spectrum being strictly inside the unit disc. We focus our attention on the region of the parameter space where the truncated normal form of the maps shows a bifurcation of a family of invariant T^1-circles into a family of invariant T^2-tori. We show that this problem needs a 3 dimensional parameter unfolding and that, for the complete maps, bifurcation occurs at points $\gamma_{\omega,\Omega}$, where ω is the rotation number on the non-normally hyperbolic T^1-circle, and $e^{\pm 2i\pi\Omega}$ are the eigenvalues of the constant matrix conjugated to the non-contracting part of the linearization on the normal fiber bundle over T^1. Making some non-resonance and diophantine assumptions on (ω, Ω) leading to a positive measure Cantor set in T^2, we show that in paraboloïdal regions of the 3 dim. parameter space we have "clean" bifurcations as for the truncated normal form. The complement of these regions forms a set of bubbles such as the ones obtained by Chenciner in [Chen] for a codimension 2 problem for maps in $\mathbb{R}^2$. The main tool here is a generalization for a matrix function on T^1, close to a constant, of the quasi-conjugacy to a constant, modulo a minimum of additional parameters ("moved" quasi-conjugacy). For the infinite dimensional case we use a C^∞ decoupling result on the angular dependent linear parts into a contraction, still angular dependent, and another part quasi-conjugated to a constant matrix. This type of analysis applies for a wide range of problems, where truncated normal forms of the maps give bifurcations from T^n to T^{n+1} tori, and this needs a $(n+1)$-dimensional parameter unfolding.