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Connecting Moduli Spaces of Calabi-Yau Threefolds

Paul S. Green[1] and Tristan Hübsch[2]*,**

[1] Department of Mathematics, University of Maryland, College Park, MD 20742, USA
[2] Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Abstract. We demonstrate that many families of Calabi-Yau threefolds consist generically of small resolutions of nodal forms in other families and, in fact, that a large class of families is connected by this relation. Our result resonates with a conjecture of Reid that Calabi-Yau threefolds may have a universal moduli space even though they are of different homotopy types. Such ideas tie quite naturally to alluring prospects of unifying (super)string models.

1. Introduction

There is a growing list of constructions of Calabi-Yau threefolds, i.e., compact complex threefolds with vanishing first Chern class. Among them, there exists a large family of non-singular Calabi-Yau threefolds embedded as complete intersections in products of complex projective spaces (CICYs) which were studied by the authors and others [1, 2]. A particularly interesting phenomenon was discussed in ref. [2], where it was called "contracting" (*vs.* its inverse, "splitting") whereby the non-singular varieties of one family of CICYs turn out generically to be small resolutions of nodal varieties in another family. Here we make a suitable generalization of this notion and study its effect on the Calabi-Yau threefolds $\mathcal{M}$ together with their deformation spaces Θ.

Clemens [3] has studied "double solids", i.e., double covers of $\mathbb{P}^3$ branched over possibly singular surfaces; in the case when the branching surface is octic and nodal, a small resolution of the double cover is a Calabi-Yau manifold. We show that in several cases CICYs are small resolutions of double covers of $\mathbb{P}^3$ branched over nodal octics, and prove similar results for the generalization of double solids to branched double covers of more general threefolds.

Reid has suggested [4] that if Θ' contracts to Θ (as will be specified below), it is reasonable to identify Θ' with a singular stratum of Θ, and conjectures the existence of a unified moduli space of (generically non-algebraic) threefolds of

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** On leave from "Ruder Bošković" Institute, Bijenička 54, YU-41000 Zagreb, Croatia, Yugoslavia