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Stochastic Schrödinger Operators and Jacobi Matrices on the Strip

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Abstract. We discuss stochastic Schrödinger operators and Jacobi matrices with wave functions, taking values in $\mathbb{C}^l$ so there are $2l$ Lyaponov exponents $\gamma_1 \geqq \cdots \geqq \gamma_l \geqq 0 \geqq \gamma_{l+1} \geqq \cdots \geqq \gamma_{2l} = -\gamma_1$. Our results include the fact that if $\gamma_1 = 0$ on a set positive measure, then V is deterministic and one that says that $\{E|\text{exactly } 2j\ \gamma\text{'s are zero}\}$ is the essential support of the a.c. spectrum of multiplicity $2j$.

1. Introduction

This paper discusses stochastic Schrödinger operators (see [4, 20, 7] for background) on $\mathbb{R}$, that is

$$H_\omega = -\frac{d^2}{dx^2} + V_\omega(x) \tag{1.1}$$

on $L^2(\mathbb{R}, dx)$, and its discrete analog:

$$(h_\omega u)(n) = u(n+1) + u(n-1) + V_\omega(n)u(n) \tag{1.2}$$

on $l^2(\mathbb{Z})$, where V_ω is a stochastic process. Several years ago, one of us (*SK*) [10] developed a set of ideas relating m-functions, the Lyaponov exponent and absolutely continuous spectrum for (1.1), and subsequently, the other of us (*B.S.*) [19] extended the ideas of [10] to equations of the form (1.2). Among the results were (γ = Lyaponov exponent):

(a_0) $\{E|\gamma(E)=0\} \equiv A$ is the essential support of $d\mu_{\mathrm{ac}}^\omega$.

(b_0) If A containts an open interval, I, then $\sigma(H_\omega)\upharpoonright I$ is purely absolutely continuous

(c_0) If $|A| > 0$ ($|\cdot|$ = Lebesgue measure), then V_ω is deterministic.

Our goal here is to discuss these results for operators on strips. The basic operator (1.2) on a strip is defined by considering a connected set $S \subset \mathbb{Z}^{\nu-1}$

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