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Beauty and the Beast: Superconformal Symmetry in a Monster Module

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Abstract. Frenkel, Lepowsky, and Meurman have constructed a representation of the largest sporadic simple finite group, the Fischer–Griess monster, as the automorphism group of the operator product algebra of a conformal field theory with central charge $c = 24$. In string terminology, their construction corresponds to compactification on a $\mathbf{Z}_2$ asymmetric orbifold constructed from the torus $\mathbf{R}^{24}/\Lambda$, where Λ is the Leech lattice. In this note we point out that their construction naturally embodies as well a larger algebraic structure, namely a super-Virasoro algebra with central charge $\hat{c} = 16$, with the supersymmetry generator constructed in terms of bosonic twist fields.

1. Introduction

A fundamental example of the techniques of conformal field theory arises in the construction of a moonshine module for the Fischer–Griess monster (the largest sporadic simple finite group) by Frenkel, Lepowsky, and Meurman (FLM) [1]. While this construction explains some of the puzzling connections between the representation theory of the monster and modular forms (referred to as moonshine in [2]), many aspects of this connection remain obscure.

In this note we point out a further feature of the construction, namely the existence of an underlying superconformal algebra in which the fermionic component of the super stress-energy tensor is constructed in terms of bosonic twist fields. In Sects. 2 and 3, we will first present a brief summary of the Virasoro and super-Virasoro algebras and their representations in terms of operator product expansions. In Sects. 4 and 5, we then present a brief synopsis of the FLM construction of the monster in the language of conformal field theory. We have tried to make these sections particularly accessible to string theorists, who may wish to satisfy their cultural curiosity in the "monstrous game." In Sect. 6 we

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