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Random Walk in Random Environment: A Counterexample?

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Abstract. We describe a family of random walks in random environments which have exponentially decaying correlations, nearest neighbor transition probabilities which are bounded away from 0, and yet are subdiffusive in any dimension $d < \infty$.

1. Introduction

Random walks in random environments have been the subject of much attention in recent years in connection with $1/f$ noise [1] and as disordered systems of interest in their own right. They have been studied by various nonrigorous methods: Monte Carlo studies [2], series expansions [3, 4], and the renormalization group [5–7]; some special models have been analyzed rigorously [8, 9]. Here we have cited only papers about the model in dimensions $d > 1$; the literature concerning the one dimensional problem is too large to catalogue.

At this point a consensus has developed [2, 3, 5–7] that for a model with short range correlations, two is the upper critical dimension for the problem: for $d > 2$ the mean square displacement will be asymptotically linear in time (i.e., normal diffusive behavior), while for $d < 2$ the behavior is subdiffusive. The point of this paper is to describe an example which casts some doubt on the universality of the last conclusion. Specifically, we describe a family of models with spatially homogeneous random environments which have exponentially decaying correlations and nearest neighbor transition probabilities bounded away from 0 so that a random walk in any of these random environments is subdiffusive in any dimension $d < \infty$.

The models we will consider are a special case of what we have called [9] random walk on a random hillside. In these systems one starts with a random

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