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Geometry of Superconformal Manifolds

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To the memory of our friend and colleague Vadim Knizhnik

Abstract. The main facts about complex curves are generalized to superconformal manifolds. The results thus obtained are relevant to the fermion string theory and, in particular, they are useful for computation of determinants of super laplacians which enter the string partition function.

Introduction

The computation of fermion string amplitudes in the Polyakov formalism [1] reduces to integration over a finite-dimensional superspace, the space of classes of superconformal manifolds [2]. Superconformal manifolds were introduced in [2], and after that in an independent paper [3] they were introduced under the name "super Riemann manifolds." (Note that we reserve the name "super Riemann manifold" for a different usage, following the terminology of [4]. Reference [4] contains also a detailed account of part of the results of [2] and of later work [8, 9].) In [2] the space of classes of superconformal manifolds was described. It was shown also that in computing the fermion string partition function a measure arose on this superconformal moduli space which was written in terms of determinants of certain operators analogous to the Laplacian. In [8] these determinants were expressed via a super analog of Selberg's zeta-function. (See [9] for details. The corresponding expression for the bosonic string was obtained first in [2].) Our purpose is now to study analytical properties of string measure on the moduli superspace and to find formulae for determinants of superlaplacians and for this measure. It is worthwhile to note that the analytical properties of the fermion string measure are essential in establishing the connection of this theory with the superstring theory of Green and Schwarz (see [12]) and with the heterotic string theory. (Note also that a supermoduli space relevant to the heterotic string was considered in [5].)

The background for our considerations will be the geometry of superconformal manifolds which is the subject of the present paper. Applications to string theory will be given in a following paper [13], the results of which are recapitulated below.