Commun. Math. Phys. 119, 75–93 (1988)

Communications in Mathematical Physics

The Quantum Theory of Second Class Constraints: Kinematics

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Abstract. The problem of second class quantum constraints is here set up in the context of C^*-algebras, utilizing the connection with state conditions as given by the heuristic quantization rules. That is, a constraint set is said to be first class if all its members can satisfy the same state condition, and second class otherwise. Several heuristic models are examined, and they all agree with this definition. Given then a second class constraint set, we separate out its first class part as all those constraints which are compatible with the others, and we propose an algebraic construction for imposition of the constraints. This construction reduces to the normal one when the constraints are first class. Moreover, the physical automorphisms (assumed as conserving the constraints) will also respect this construction. The final physical algebra obtained is free of constraints, gauge invariant, unital, and with the right choice, simple. This C^*-algebra also contains a factor algebra of the usual observables, i.e. the commutator algebra of the constraints. The general theory is applied to two examples—the elimination of a canonical pair from a boson field theory, as in the two dimensional anomalous chiral Schwinger model of Rajaraman [14], and the imposition of quadratic second class constraints on a linear boson field theory.

1. Introduction

The classical treatment of degenerate systems by the Dirac procedure [1] has reached a high degree of mathematical maturity in the symplectic formulation of Gotay, Nester and Hinds [2]. As for the quantum setting, these systems still remain within heuristic formulations [3] without drawing much from their classical rigor, due to the dubious nature of quantization [4]; geometric quantization [5] as yet being only a prequantization. It is our aim here and in our previous papers [6–8] to give a quantum mechanical procedure for eliminating degeneracy in a mathematically consistent way, and we do this in a C^*-algebra framework.