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On Static and Radiative Space-Times

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Abstract. The conformal constraint equations on space-like hypersurfaces are discussed near points which represent either time-like or spatial infinity for an asymptotically flat solution of Einstein's vacuum field equations. In the case of time-like infinity a certain "radiativity condition," is derived which must be satisfied by the data at that point. The case of space-like infinity is analysed in detail for static space-times with non-vanishing mass. It is shown that the conformal structure implied here on a slice of constant Killing time, which extends analytically through infinity, satisfies at spatial infinity the radiativity condition. Thus to any static solution exists a certain "radiative solution" which has a smooth structure at past null infinity and is regular at past time-like infinity. A characterization of these solutions by their "free data" is given and non-symmetry properties are discussed.

1. Introduction

In article [9] the hyperboloidal initial value problem for Einstein's field equations was introduced. Here the prospective solution space-time is envisioned as having a smooth structure at future null infinity and data are given such that they represent the first and second fundamental forms on a space-like hypersurface which intersects null infinity in a space-like 2-sphere. In [12] has been shown that such "hyperboloidal initial data," if they are sufficiently close to Minkowskian hyperboloidal initial data, evolve into a solution of Einstein's equations which is future null geodesically complete, has in the future of the initial surface a smooth structure at null infinity, and is regular at future time-like infinity in the sense that in a suitable conformal extension of the solution a point i^+ exists, representing future time-like infinity, such that the past directed null geodesics through i^+ generate future null infinity for the solution space-time.

This result reduces the question whether there exist non-trivial "purely radiative space-times," that is smooth solutions of Einstein's vacuum field equations which have a smooth and complete structure at past null infinity $\mathscr{I}^-$ and future null infinity $\mathscr{I}^+$ and which are regular at past time-like infinity i^- and future time-like infinity i^+, to the analysis of the behaviour of the solutions of the standard Cauchy problem near spatial infinity. In the following spatial infinity will be thought of