Commun. Math. Phys. 119, 29-50 (1988)

Communications in Mathematical Physics

Geometric Invariants of the Quantum Hall Effect

Jingbo Xia*

Department of Mathematics, State University of New York at Buffalo, Buffalo, New York 14214, USA

Abstract. We study the two-dimensional Hall effect with a random potential. The Hall conductivity is identified as a geometric invariant associated with an algebra of observables. Using the pairing between K-theory and cyclic cohomology theory, we identify this geometric invariant with a topological index, thereby giving the Hall conductivity a new interpretation.

Introduction

Since its experimental discovery [21], the quantum Hall effect [23] has attracted the attention of many researchers. The general approach in the theoretical work is that one considers a one-particle theory in a two-dimensional sample of infinite size. This leads to the study of the Hamiltonian

$$H_\omega = [K_x^2 + K_y^2]/2 + Q_\omega(x, y), \quad \omega \in \Omega, \tag{0.1}$$

with the commutation relation

$$[K_x, K_y] = i\beta.$$

Here, β accounts for the presence of a magnetic field and Q is a real function on the state space Ω which admits a flow of translation by $\mathbb{R}^2$. D. Thouless *et al.* [28] considered periodic potentials Q, i.e. the case where Ω is the two dimensional torus $T^2 = \mathbb{R}^2/\mathbb{Z}^2$ with the natural flow induced by $\mathbb{R}^2$. They showed that with such a potential and a magnetic field of rational flux, the conductance of any filled, isolated band in the spectrum of the Hamiltonian is a topological invariant which is an integral multiple of $e^2/\hbar$. Since then substantial progress has been made on this subject through the contributions of many authors [1, 2, 3, 5–8, 18, 22]. Among these references, J. Bellissard's work has a close relation with this paper.

It was first observed in [5] that the two-dimensional Hall conductivity can be identified as the Chern character of the spectral projection of H_ω corresponding to $(-\infty, E_F)$, where E_F is the Fermi level. This led to the introduction of A. Connes' geometry-operator algebra techniques to the study of the quantum Hall effect

---

* Supported in part by the National Science Foundation under Grant No. DMS-8717185