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Regularity of the Scattering Trajectories in Classical Mechanics

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Abstract. We establish conditions under which all trajectories of a mechanical system have a regular scattering behavior. Applications to the many-body problems on the line and to the systems with exponential cone potentials are worked out.

1. Introduction

To motivate our study let us consider the classical motion of n d-dimensional particles with pairwise interactions. We consider the situation where the forces of interaction are central, conservative and repulsive. Denote by $x_1,\dots,x_n$ the positions and by $\dot{x}_1,\dots,\dot{x}_n$ the velocities of the particles. Let $m_1,\dots,m_n$ be their masses and $v_{ij} \geqq 0$, $1 \leqq i < j \leqq n$, the potentials of interaction. Then the total energy is

$$E(x,\dot{x}) = \frac{1}{2}\sum_{i=1}^{n} m_i \|\dot{x}_i\|^2 + \sum_{i<j} v_{ij}(\|x_i - x_j\|). \tag{1.1}$$

It is known [1, 9] that any motion $\{x_1(t),\dots,x_n(t);\ -\infty < t < \infty\}$ has asymptotic velocities at infinity

$$\dot{x}_i(\infty) = \lim_{t\to\infty} \dot{x}_i(t), \quad i = 1,\dots,n. \tag{1.2}$$

Assume that the pair potentials satisfy the fast decay condition

$$\int_{\rho}^{\infty} v_{ij}(r)\,dr < \infty. \tag{1.3}$$

It was shown in [2] that the motions with distinct asymptotic velocities

$$\dot{x}_1(\infty) \neq \cdots \neq \dot{x}_n(\infty) \tag{1.4}$$

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