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Classical and Quantum Scattering on a Cone*

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Abstract. The "topological" scattering of a quantized test particle in the locally flat conical geometry of a localized source in $2+1$-dimensional gravity is analyzed. Wave functions and scattering amplitudes are obtained and compared with those recently found in a different approach by 't Hooft. The propagator [heat kernel] is also determined.

I. Introduction

Einstein theory in $2+1$ dimensions is an appealing model for investigating the effects of quantized gravity because the full curvature is locally determined by matter. Consequently, there are no gravitational degrees of freedom and no gravitational interaction among localized sources since space-time is flat between them; dynamics is topology. As a preliminary, we investigated the properties of the classical theory some time ago [1–3] following earlier work [4]. There is also a close relation to cosmic strings in four dimensions since the space-time of an infinite straight string is effectively three-dimensional [5].

Recently, 't Hooft [6] has begun consideration of the quantum theory in terms of the interaction among quantized particle sources [gravity itself is of course only "quantized" through the sources]. In particular, he has analyzed two-particle scattering by examining the relative motion, which he reduces to that of a particle moving on a cone. In the present work, we shall treat the closely related problem: scattering of a test particle in the field of a stationary point mass, which gives rise to a locally flat, conical metric [1, 3, 4]. Hence we also need to find solutions of the Schrödinger equation on a cone. To this end, we use a partial wave expansion, adapted to the fact that there is no potential; the kinetic term carries the information that our space has conical geometry.

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