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A New Method for the Thermomodynamics of the BCS Model

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Abstract. Using large deviations in combination with the Berezin-Lieb inequalities, we analyse the phase-transition in the BCS model with non-constant energies and interactions.

1. Introduction

N.N. Bogoliubov and his school have made several attempts at solving the full BCS model with hamiltonian

$$H = -\sum_{\mathbf{k}, s=\pm 1} \varepsilon(\mathbf{k}) a^*_{\mathbf{k},s} a_{\mathbf{k},s} - \frac{1}{V} \sum_{\mathbf{k}} \sum_{\mathbf{k}'} a^*_{-\mathbf{k},-1} a^*_{\mathbf{k},1} U(k,k') a_{\mathbf{k}',1} a_{-\mathbf{k}',-1} \tag{1.1}$$

(see [1] and the references therein). The first of these was by a perturbation expansion which can be made rigorous only at zero temperature. Later they developed a mini-max principle which allowed them to treat a class of interactions. In this paper we provide a new method for treating BCS-type models in the quasi-spin formulation

$$H = -\sum_{\mathbf{k}} \varepsilon(\mathbf{k}) \sigma^z_{\mathbf{k}} - \frac{1}{V} \sum_{\mathbf{k}} \sum_{\mathbf{k}'} \sigma^+_{\mathbf{k}} U(\mathbf{k}, \mathbf{k}') \sigma^-_{\mathbf{k}'} \ , \tag{1.2}$$

which can be applied to more general interactions. The result in this paper have already been announced without proof in [2] and an extension of the method to treat this type of model in the original formulation of (1.1) will be given in [3]. The techniques developed here have also been used for the full spin-boson model [4].

Recently Cegła, Lewis, and Raggio [7] have been able to obtain the free energy density for quantum spin systems with homogeneously decomposable hamiltonians. Their methods, amongst other things, allow them to streamline the treatment of the thermodynamics of the BCS model [5, 6] in the strong coupling limit in which $\varepsilon(\mathbf{k})$ and $U(\mathbf{k},\mathbf{k}')$ are replaced by their average value and of other models whose hamiltonians are functions of the total spin operators. They obtained a large deviation principle [8, 9] for the measures arising from the multiplicities of