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Nonlinear Stability of Rarefaction Waves for Compressible Navier Stokes Equations

Tai-Ping Liu[1]* and Zhouping Xin[2]**

[1] Department of Mathematics, University of Maryland, College Park, MD 20742, USA

[2] Department of Mathematics, University of Michigan, Ann Arbor, MI 48109, USA

Abstract. It is shown that expansion waves for the compressible Navier–Stokes equations are nonlinearly stable. The expansion waves are constructed for the compressible Euler equations based on the inviscid Burgers equation. Our result shows that Navier–Stokes equations and Euler equations are time-asymptotically equivalent on the level of expansion waves. The result is proved using the energy method, making essential use of the expansion of the underlining nonlinear waves and the specific form of the constitutive eqution for a polytropic gas.

1. Introduction

Consider one-dimensional compressible Navier–Stokes equations in the Lagrangian Coordinates,

$$v_t - u_x = 0, \tag*{(1.1)$_1$}$$

$$u_t + p_x = (\mu u_x(v))_x, \tag*{(1.1)$_2$}$$

$$\left(e + \frac{u^2}{2}\right)_t + (pu)_x = (\kappa\theta_x/v + \mu u u_x/v)_x, \tag*{(1.1)$_3$}$$

where v, u, p, e and θ are, respectively, the specific volume, velocity, pressure, internal energy and the temperatures of the gas, and the positive constants μ, κ are the viscosity and heat conductivity coefficients. The gas is assumed to be polytropic:

$$p = R\theta/v = A\exp(s/c_v)v^{-\gamma}, \quad e = \frac{R\theta}{\gamma - 1}, \tag{1.2}$$

where s is the entropy, $R > 0$ the gas constant, $\gamma > 1$ the adiabatic constant, $c_v = R/(\gamma - 1)$ the specific heat, and A a positive constant. We are interested in the

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