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Extension of the Pirogov–Sinai Theory to a Class of Quasiperiodic Interactions*

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Abstract. We extend the Pirogov–Sinai theory in such a manner that it applies to a large class of models with small quasiperiodic interactions as perturbations of periodic ones. We find general diophantine conditions on the frequency module of the quasiperiodic interactions and derivability conditions on the interaction potentials. These conditions allow to prove that the low temperature phase diagram is a homeomorphic deformation of the phase diagram at zero temperature.

1. Introduction and Main Results

The standard Pirogov–Sinai theory studies the low temperature phase diagram for discrete spin systems on a lattice described by hamiltonians with finite range of interactions which are translational invariant or periodic. Namely, for hamiltonians having a finite $(m+1)$ number of constant or periodic ground states and satisfying the Peierls condition, the Pirogov–Sinai theory asserts that the topological structure of the phase diagram at sufficiently low temperature is the same as that of the diagram at zero temperature [15] (see [18, 19] for pedagogical expositions).

In order to study the structure of the phase diagram, the original hamiltonian H_0 which is assumed to have exactly $m+1$ ground states, is perturbed by small additional terms associated to a vector of coupling constants $\xi = (\xi_1, \dots, \xi_m) \in \mathbb{R}^m$. For example one could consider

$$H_\xi = H_0 + \sum_{i=1}^{m} \xi_i H_i$$

(non-linear dependence of H_ξ is also possible). One requires the perturbations H_i to remove the degeneracy of the hamiltonian H_0. The phase diagram at zero temperature is obtained by minimizing H_ξ in the space of the parameters ξ. In general, this phase diagram has the following topological structure: there is a point

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