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Etiology of IRF Models

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Abstract. We show that a class of $2D$ statistical mechanics models known as IRF models can be viewed as a subalgebra of the operator algebra of vertex models. Extending the Wigner calculus to quantum groups, we obtain an explicit intertwiner between two representations of this subalgebra.

Two major progresses have recently been made in the understanding of two dimensional lattice models. The first is the classification of rational and trigonometric solutions of the Yang-Baxter equations for a class of models known as vertex models [1–5]. The second is the discovery of many representatives of another class known as IRF (interacting round a face) models and their study in connection with conformal field theories [6–11]. In the appendix of [11] it was shown that both classes correspond to different representations of the same algebra, and it is the aim of this letter to complete the identification by building an explicit intertwiner between them. The method we use is an application to the quantum group case of Ocneanu's cell technique [12].

Quantum Wigner Calculus. Consider the associative algebra $\hat{U}(SU(2))$ [4, 5] generated by the symbols $q^{h/2}$, J_+, J_- under the following relations:

$$q^{h/2}q^{-h/2}=q^{-h/2}q^{h/2}=1\ , \qquad q^{h/2}J_+q^{-h/2}=qJ_+\ , \qquad q^{h/2}J_-q^{-h/2}=q^{-1}J_-\ ,$$

$$[J_+J_-]=\frac{q^h-q^{-h}}{q-q^{-1}}\ . \tag{1}$$

We denote by $\Delta^{(N)}$ the coproduct homomorphism $\Delta^{(N)}\hat{U}\to\hat{U}\otimes^N$ (N fold tensor product)

$$\Delta^{(N)}(q^{h/2})=q^{h/2}\otimes q^{h/2}\ldots\otimes q^{h/2}\ ,$$

$$\Delta^{(N)}(J_\pm)=\sum_{v=1}^{N} q^{h/2}\otimes\ldots\otimes q^{h/2}\overset{v}{\otimes J_\pm}\otimes q^{-h/2}\ldots\otimes q^{-h/2}\ . \tag{2}$$