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The Free Energy of Quantum Spin Systems and Large Deviations

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Abstract. For a quantum system of n identical spins of magnitude j, we introduce an integrated density of states of definite total spin angular momentum. The underlying sequence $\{\mathbb{K}_n^j : n=1,2,\ldots\}$ of probability measures satisfies Varadhan's large deviation principle, and converges to a degenerate distribution. We use the Berezin-Lieb Inequalities to obtain upper and lower bounds for the limiting specific free-energy of the spins interacting with a second quantum system under specified conditions on the Hamiltonian. The method is illustrated by applications to the BCS model and to the Dicke maser model.

1. Introduction

Large-deviation methods are proving useful in statistical mechanics [1, 2], both for reorganizing existing proofs and for obtaining new results [3–5]. The main purpose of this paper is to prove a large deviation result which has applications to the equilibrium statistical mechanics of spin systems; we illustrate its use by applying it to the calculation of the specific free energy of the BCS model and of the Dicke Maser model. In the case of the BCS model, it provides a short proof of a known result; in the case of the Dicke Maser model, when combined with the Berezin-Lieb inequalities, it provides a substantial improvement on existing treatments in that it requires minimal restrictions on the interaction terms. In both cases, the spins are not required to be of magnitude 1/2.

The Laplace method (the method of the largest term) is at the heart of equilibrium statistical mechanics, but to make it rigorous in particular instances is usually tedious and sometimes subtle. The large deviation principle, in Varadhan's formulation [6], provides an efficient way of supplying the required proofs; it reduces the tedium and exposes any latent subtleties. The abstract setting is that of a sequence $\{\mathbb{K}_n : n=1,2,\ldots\}$ of probability measures on the Borel subsets of a complete separable metric space E (in the main theorem of this paper, E is the interval $[0,1]$), and a divergent sequence $\{V_n : n=1,2,\ldots\}$ of positive numbers. We