

An Intermediate Phase with Slow Decay of Correlations in One Dimensional $1/|x - y|^2$ Percolation, Ising and Potts Models

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Abstract. We rigorously establish the existence of an intermediate ordered phase in one-dimensional $1/|x - y|^2$ percolation, Ising and Potts models. The Ising model truncated two-point function has a power law decay exponent θ which ranges from its low (and high) temperature value of two down to zero as the inverse temperature and nearest neighbor coupling vary. Similar results are obtained for percolation and Potts models.

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