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Line Bundles on Super Riemann Surfaces

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Abstract. We give the elements of a theory of line bundles, their classification, and their connections on super Riemann surfaces. There are several salient departures from the classical case. For example, the dimension of the Picard group is not constant, and there is no natural hermitian form on Pic. Furthermore, the bundles with vanishing Chern number aren't necessarily flat, nor can every such bundle be represented by an antiholomorphic connection on the trivial bundle. Nevertheless the latter representation is still useful in investigating questions of holomorphic factorization. We also define a subclass of all connections, those which are compatible with the superconformal structure. The compatibility conditions turn out to be constraints on the curvature 2-form.

1. Introduction

This paper is a sequel to [1, 2]. In those papers we described the theory of super Riemann surfaces (SRS) in differential-geometric terms.[1] In particular we defined a SRS $\hat{X}$ as a supermanifold of real dimension $2|2$ equipped with an additional structure. This "superconformal structure" amounts to an integrable reduction of the structure group of $\hat{X}$. $\hat{X}$ then has a canonical holomorphic line bundle $\hat{\omega}$, so we can define holomorphic $\frac{p}{2}$-differentials as sections of $\hat{\omega}^p$. We also get an analog of the Cauchy-Riemann operator, $\hat{\partial}$, which can be used to define both the string action and actions for generalized first-order systems [3]. All in all, SRS show a remarkable formal similarity to ordinary Riemann surfaces, despite the fact that they cannot be thought of as having just one complex dimension.

In this paper we will carry the discussion further, turning to other structures on Riemann surfaces and their SRS analogs. We begin by reviewing the basic

[1] See the references in [2]