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An Instanton-Invariant for 3-Manifolds

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Abstract. To an oriented closed 3-dimensional manifold M with $H_1(M,\mathbb{Z})=0$, we assign a $\mathbb{Z}_8$-graded homology group $I_*(M)$ whose Euler characteristic is twice Casson's invariant. The definition uses a construction on the space of instantons on $M\times\mathbb{R}$.

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1. Instanton Homology

1a) Introduction

Let M be a closed connected oriented 3-manifold. As is well known (see e.g. [He]), every 3-dimensional topological manifold carries a unique differentiable structure, so that we can consider M in either of these two categories. For the sake of brevity, we will refer to M simply as a 3-manifold.

A strong algebraic invariant of M is its fundamental group $\pi_1(M)$. Unfortunately, as a satisfactory description of the set 3-manifolds, the fundamental group falls short in two crucial ways: First, the classification of manifolds with isomorphic fundamental groups depends on the well known and as yet unsettled