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On Generally Covariant Quantum Field Theory and Generalized Causal and Dynamical Structures

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Abstract. We give an example of a generally covariant quasilocal algebra associated with the massive free field. Maximal, two-sided ideals of this algebra are algebraic representatives of external metric fields. In some sense, this algebra may be regarded as a concrete realization of Ekstein's ideas of presymmetry in quantum field theory. Using ideas from our example and from usual algebraic quantum field theory, we discuss a generalized scheme, in which maximal ideals are viewed as algebraic representatives of dynamical equations or Lagrangians. The considered frame is no quantum gravity, but may lead to further insight into the relation between quantum theory and space-time geometry.

1. Introduction

One of the most fascinating challenges of contemporary physics is the unification of Einstein's general relativity theory with quantum theory. Many attempts are made, but the goal seems to be still far off. It even is not clear how much of the conceptual and technical structures of both theories will survive an unification, because they seem to be fundamentally different.

By results of [1, 3, 5, 7, and 8], there emerge ideas of a quantum field theory which incorporates at least one of the basic principles of general relativity theory: the *principle of general covariance*. The setting is algebraic quantum field theory which seems to be especially suitable.

In algebraic quantum field theory (cf. Haag and Kastler [10]) to each region (open set with compact closure) $\mathcal{O}$ of Minkowski space $\mathbb{M}$ there corresponds one C^*-algebra $\mathcal{A}(\mathcal{O})$. This correspondence is assumed to fullfill the isotony property, i.e., if $\mathcal{O}_1 \subseteqq \mathcal{O}_2 \subseteqq \mathbb{M}$, then $\mathcal{A}(\mathcal{O}_1) \subseteqq \mathcal{A}(\mathcal{O}_2)$. The self-adjoint elements of $\mathcal{A}(\mathcal{O})$ are interpreted as observables that detect events within $\mathcal{O}$. All $\mathcal{A}(\mathcal{O})$ are subalgebras of a C^*-algebra $\mathcal{A}$ which is the inductive limit of all the $\mathcal{A}(\mathcal{O})$. $\mathcal{A}$ is also called *algebra of quasilocal observables*, and the correspondence $\mathcal{O} \to \mathcal{A}(\mathcal{O})$ *net of local C^*-algebras*. Among the (Haag-Kastler) axioms of algebraic quantum field theory