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Non-Perturbative 2 Particle Scattering Amplitudes in 2+1 Dimensional Quantum Gravity

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Abstract. A quantum theory for scalar particles interacting only gravitationally in $2+1$ dimensions is considered. Since there are no real gravitons the interaction is entirely topological. Nevertheless, there is non-trivial scattering. We show that the two-particle amplitude can be computed exactly. Although the complete "theory" is not well understood we suggest an approach towards formulating the N particle problem.

1. Introduction

It is not known how to quantize gravity without running into infinity problems or topological contradictions such as the ones that are hampering our understanding of black holes. Even in $2+1$ dimensions quantum gravity is non-renormalizable. Yet there is reason to hope that a consistent formulation of a quantum theory can be given that yields classical $2+1$ dimensional gravity in the limit $\hbar \Rightarrow 0$. Our reason for thinking this is that in $2+1$ dimensions the gravitational interaction is entirely topological; there are no real gravitons, and the only degrees of freedom are whatever other particles are being introduced. Classically, the "interaction" is simple and beautiful [1]: every particle is surrounded by a space-time in the form of a cone. The conical singularity is at the world line of the particle, and the deficiency angle at this singularity can be defined to be equal to the particle's mass (we put Newton's constant equal to one). As a consequence, two particles passing each other at the right proceed in a direction slightly different from the one they choose when they pass each other at the left.

If we know for each particle at which side they pass each other particle then the classical scattering process is trivial to compute: they all continue in straight lines. If this is so simple, why then can't we "quantize" this system by attributing wave packets to these particles?

Trying to do just this, one discovers a difficulty. Particles in the wave packets are not well localized. This does *not* stop us from writing down one-particle wave equations on a cone, but the difficulty comes in writing down *many*-particle wave equations. Where exactly is the conical singularity produced by one particle in the space-time of another?