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Renormalization of Lattice Feynman Integrals with Massless Propagators

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Abstract. A renormalization procedure is proposed which applies to lattice Feynman integrals containing zero-mass propagators and is analogous to the BPHZL renormalization procedure for continuum Feynman integrals. The renormalized diagrams are infrared convergent for non-exceptional external momenta, if the vertices of the theory satisfy a general infrared constraint. Under the same conditions as in the massive case [4], the continuum limit of the renormalized theory exists and is independent of the details of the lattice action.

1. Introduction

Feynman integrals with a lattice cutoff have a very specific structure. They are absolutely convergent for finite lattice spacing, if all propagators are massive. The continuum limit behavior of such diagrams is described by a lattice power counting theorem [3], which uses a new kind of an ultraviolet (UV) divergence degree (the well known power counting theorems of Weinberg [1] and of Hahn and Zimmermann [2] do not apply to diagrams with a lattice cutoff). On the basis of such a power counting theorem a renormalization program for lattice field theories has been given [4], which is analogous to the BPHZ finite part prescription for continuum Feynman integrals [5].

These methods work for massive field theories. In the presence of massless fields, additional arguments are needed to avoid infrared (IR) divergencies. It has been shown [6] that the UV-power counting conditions only have to be supplemented by IR-power counting conditions, and IR-singularities are tractable by the same methods as in the continuum [7, 8]. In this article, we use this power counting to give a renormalization procedure for lattice Feynman integrals with massless propagators.

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