Commun. Math. Phys. 117, 625–638 (1988)

Communications in
Mathematical
Physics

Initial Data Sets of Minimal Energy

Oscar Reula

Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1,
D-8046 Garching bei München, Federal Republic of Germany

Abstract. The total mass of a gravitational system, the ADM mass, is considered as a functional of the initial data sets for sources and gravitation. From the condition that such a functional has a minimum under a restricted class of variations, tensorial equations for the gravitational initial data are obtained. The solutions to these equations, whose existence for a large class of source fields is asserted, represent initial data sets for gravitation satisfying the constraint equations and may be considered as having no gravitational radiation.

1. Introduction

An important problem in general relativity is to compute the gravitational radiation output of isolated systems. In principle this can be done by prescribing in a hypersurface S initial data for the source and the gravitational fields, evolve them according to Einstein and source field equations, and finally evaluate the News function at the $\mathscr{I}^+$ thus generated. Besides the usual problems arising from evolving for an infinite time non-linear equations, the above program suffers from another one, namely of determining which are the appropriate data to be given at S in order that the radiation we register at $\mathscr{I}^+$ is the one generated by the source motion, for it is intuitively clear that, given any compact source field initial data one can find different initial data sets for the gravitational field such that:

i) They are consistent with the source data, in the sense that the constraint equations are satisfied, ii) the resulting source evolution is almost identical during some finite interval, but iii) the amount of radiation produced at $\mathscr{I}^+$ is radically different.

Due to the interaction between the source and the gravitational fields the concept of *the radiation produced by the source* is ambiguous. Thus, there exist several procedures determining different initial data sets which pretend to capture this concept [1]. Some intend to compute in successive approximations a solution to the whole space-time and so, in pushing the problem to $\mathscr{I}^-$, where at least we