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A Presentation for the Virasoro and Super-Virasoro Algebras

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Abstract. It is shown that the entire Virasoro, Ramond and Neveu–Schwarz algebras can each be constructed from a finite number of well-chosen generators satisfying a small number of conditions. The most economical sets consist of just two starting generators in all cases, subject to eight conditions for the Virasoro case, five conditions for the Ramond case, and nine conditions for the Neveu–Schwarz case.

Introduction

The Virasoro algebra [1, 2], for example

$$[L_m, L_n] = (m-n)L_{m+n} + c(m^3 - m)\delta_{m+n,0}, \tag{1}$$

(where the indices m and n run over the positive and negative integers) is normally contrasted to the classical algebras, as their closure contains only a finite number of generators. In one respect, however, the Virasoro algebra resembles these algebras, even down to their simple prototype $SU(2)$: a small number of generators, in this case 2, suffices to define the rest through appropriate chains of commutations. The entire algebra then follows inductively by use of the Jacobi identities, provided that a finite number of independent commutation relations be imposed in each case. Alternatively, the structure of the entire algebra is determined as a solution of these commutation conditions, regarded as equations for the starting generators. For $SU(2)$, this is evident; given the two starting generators T_1 and T_2, the third generator T_3 is defined by

$$\text{(Def.)} \quad T_3 = -i[T_1, T_2], \tag{2}$$

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