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Finite Dimensional Representations of the Quantum Analog of the Enveloping Algebra of a Complex Simple Lie Algebra

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Abstract. Let $\mathcal{G}$ be a complex simple Lie algebra. We show that when t is not a root of 1 all finite dimensional representations of the quantum analog $U_t\mathcal{G}$ are completely reducible, and we classify the irreducible ones in terms of highest weights. In particular, they can be seen as deformations of the representations of the (classical) $U\mathcal{G}$.

I. Introduction

To each complex simple Lie algebra $\mathcal{G}$, Jimbo associates the quantum analog of its enveloping algebra, let $U_t\mathcal{G}$, where t is a non-zero parameter, as follows (see also Drinfeld [2, 3]):

Let $(a_{ij})_{1 \leqq i,j \leqq N}$ be the Cartan matrix of $\mathcal{G}$ and $(\alpha_i)_{1 \leqq i \leqq N}$ a basis of simple roots; $U_t\mathcal{G}$ is the C-algebra generated by $(k_i^{\pm 1}, e_i, f_i)_{1 \leqq i \leqq N}$ with relations:

$$k_i \cdot k_i^{-1} = k_i^{-1} \cdot k_i = 1; \quad k_i k_j = k_j k_i,$$

$$k_i e_j k_i^{-1} = t_i^{a_{ij}} e_j; \quad k_i f_j k_i^{-1} = t_i^{-a_{ij}} f_j,$$

$$[e_i, f_j] = \delta_{ij} \frac{k_i^2 - k_i^{-2}}{t_i^2 - t_i^{-2}},$$

$$\sum_{v=0}^{1-a_{ij}} (-1)^v \begin{bmatrix} 1-a_{ij} \\ v \end{bmatrix}_{t_i^2} e_i^{1-a_{ij}-v} e_j e_i^v = 0 \quad \text{for } i \neq j,$$

$$\sum_{v=0}^{1-a_{ij}} (-1)^v \begin{bmatrix} 1-a_{ij} \\ v \end{bmatrix}_{t_i^2} f_i^{1-a_{ij}-v} f_j f_i^v = 0 \quad \text{for } i \neq j,$$

where $t_i = t^{(\alpha_i|\alpha_i)/2}$, $(\ |\ )$ being the invariant inner product on $\oplus \mathbf{C}\alpha_i$, with $(\alpha_i|\alpha_i) \in \mathbf{Z}$.

$$\begin{bmatrix} m \\ n \end{bmatrix}_t = \begin{cases} \dfrac{(t^m - t^{-m})(t^{m-1} - t^{-(m-1)}) \cdots (t^{m-n+1} - t^{-(m-n+1)})}{(t - t^{-1})(t^2 - t^{-2}) \cdots (t^n - t^{-n})} & \text{for } m > n > 0, \\ 1 \quad \text{for } n = 0 \text{ or } m = n. \end{cases}$$