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Dimension Formula for Random Transformations*

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Abstract. We consider compositions of random diffeomorphisms and show that the dimension of sample measures equals Lyapunov dimension as conjectured in the nonrandom case by Yorke et al.

The role of *dimension* has long been recognized in the study of chaos (see [ER]). Roughly speaking, the dimension of a set measures the amount of information necessary to specify points within it accurately. In practice it tells us how many variables we should use to parametrize the set [Man]. This information is particularly useful for dealing with attractors of relatively low dimension embedded in a high, possibly infinite, dimensional phase space.

Various methods of experimentally computing dimension have been devised. A widely used procedure is to estimate dimension via Lyapunov exponents, computing a quantity called *Lyapunov dimension* introduced by Yorke et al [FKYY]. The validity of this procedure relies on Yorke's conjecture, a version of which states that "typically" Lyapunov dimension equals the usual notions of dimension. This is true for attractors on surfaces [Y] but when the phase space has dimension greater than 2, Yorke's conjecture has not been mathematically verified.

In this paper we do not deal with the conjecture in [FKYY] exactly as it is stated. Instead, we assume that our dynamical system is subjected to certain types of stochastic noise. We prove under this assumption that Yorke's conjectured formula for dimension is indeed mathematically correct. The model we use consists of composing random diffeomorphisms with some conditions to guarantee genuine randomness. We prove that in this setting Lyapunov dimension is equal to the dimension of the sample measures, i.e. the natural invariant family of measures associated with individual realizations of the random process. Our results are applicable to flows arising from stochastic differential equations.

Introduction

Consider first the nonrandom case. Let $f: M \circlearrowleft$ be a diffeomorphism of a compact Riemannian manifold preserving an ergodic Borel probability measure

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