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Particle Structure Analysis of Soliton Sectors in Massive Lattice Field Theories

P. A. Marchetti

Dipartimento di Fisica dell'Universitá di Padova, Via Marzolo 8, I-35131 Padova, Italy

Abstract. We discuss the particle structure in the soliton sectors of massive lattice field theories by means of convergent cluster expansions. In several models we prove that the soliton field operator with lowest charge couples the vacuum to a stable one-particle state, in a suitable region of the coupling parameter space. Both local and stringlike solitons are analyzed. We also show that the mass of the local soliton equals the surface tension.

1. Introduction

A procedure to construct quantum solitons in lattice field theories has been proposed in paper I (=[1]). The basic idea is to apply an Osterwalder-Schrader (O.S.) reconstruction theorem [2] to mixed order-disorder correlation functions.

We now shortly describe how mixed order-disorder correlation functions are constructed and how they are related to solitons.

The expectation value of a disorder field, $D(\omega)$, is given by

$$\langle D(\omega)\rangle \equiv \lim_{\Lambda\uparrow \mathbf{Z}^d_{1/2}} \frac{Z_\Lambda(\omega)}{Z_\Lambda}, \tag{1.1}$$

where ω is an external hyper-gauge field with values in a discrete abelian group $\mathscr{X}$; Z_Λ is the partition function of the theory in a lattice $\Lambda\subset \mathbf{Z}^d_{1/2}$ and $Z_\Lambda(\omega)$ denotes the partition function of the model coupled to the external field ω.

The prefix hyper-means that ω has a rank[1] higher than the ranks of the basic fields of the theory. For example, in the models discussed here, ω has rank 1 in the scalar and fermion theories (i.e. it is a true lattice gauge field) and rank 2 in gauge theories.

Assume that $\langle D(\omega)\rangle$ depends on the curvature, $d\omega$, of ω and the support of $(d\omega)^*$, the dual of $d\omega$, is given by a finite set of points $\{x_i\}_{i=1}^n$ with $(d\omega)^*(x_i)=q_i$, then $\langle D(\omega)\rangle$ will be denoted by $\langle D(x_1,q_1,\dots,x_n,q_n)\rangle$.

[1] A lattice field of rank k with values in a space W is a map from k-dimensional cells of the lattice to W