Commun. Math. Phys. 117, 451–500 (1988)

Communications in
Mathematical
Physics

Isospectral Hamiltonian Flows in Finite and Infinite Dimensions

I. Generalized Moser Systems and Moment Maps into Loop Algebras*

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Abstract. A moment map $\tilde{J}_r: \mathscr{M}_A \to (\widetilde{gl(r)}^+)^*$ is constructed from the Poisson manifold $\mathscr{M}_A$ of rank-r perturbations of a fixed $N \times N$ matrix A to the dual $(\widetilde{gl(r)}^+)^*$ of the positive part of the formal loop algebra $\widetilde{gl(r)} = gl(r) \otimes \mathbb{C}[[\lambda, \lambda^{-1}]]$. The Adler-Kostant-Symes theorem is used to give hamiltonians which generate commutative isospectral flows on $(\widetilde{gl(r)}^+)^*$. The pull-back of these hamiltonians by the moment map gives rise to commutative isospectral hamiltonian flows in $\mathscr{M}_A$. The latter may be identified with flows on finite dimensional coadjoint orbits in $(\widetilde{gl(r)}^+)^*$ and linearized on the Jacobi variety of an invariant spectral curve X_r which, generically, is an r-sheeted Riemann surface. Reductions of $\mathscr{M}_A$ are derived, corresponding to subalgebras of $gl(r, \mathbb{C})$ and $sl(r, \mathbb{C})$, determined as the fixed point set of automorphism groupes generated by involutions (i.e., all the classical algebras), as well as reductions to twisted subalgebras of $\widetilde{sl(r, \mathbb{C})}$. The theory is illustrated by a number of examples of finite dimensional isospectral flows defining integrable hamiltonian systems and their embeddings as finite gap solutions to integrable systems of PDE's.

1. Introduction

In 1979 Moser [32] showed that a number of well-known completely integrable finite dimensional hamiltonian systems could be uniformly understood in the framework of certain rank 2 isospectral deformations of matrices. The problem he considered involved hamiltonian flow $(x(t), y(t))$ in $\mathbb{R}^{2n}$ which, for a fixed $n \times n$ matrix A and real constants, a, b, c, d, leaves the spectrum of the matrix

$$L = A + ax \otimes x + bx \otimes y + cy \otimes x + dy \otimes y$$

invariant. Among the results he obtained were:

* This research was partially supported by NSF grants MCS-8108814 (A03), DMS-8604189, and DMS-8601995