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Topological Quantum Field Theory

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Abstract. A twisted version of four dimensional supersymmetric gauge theory is formulated. The model, which refines a nonrelativistic treatment by Atiyah, appears to underlie many recent developments in topology of low dimensional manifolds; the Donaldson polynomial invariants of four manifolds and the Floer groups of three manifolds appear naturally. The model may also be interesting from a physical viewpoint; it is in a sense a generally covariant quantum field theory, albeit one in which general covariance is unbroken, there are no gravitons, and the only excitations are topological.

1. Introduction

One of the dramatic developments in mathematics in recent years has been the program initiated by Donaldson of studying the topology of low dimensional manifolds via nonlinear classical field theory [1, 2]. Donaldson's work uses heavily the self-dual Yang-Mills equations, which were first introduced by physicists [3], and depends on some important results originally obtained by mathematical physicists, e.g. Taubes' theorem on existence of instantons on certain smooth four manifolds [4] (as well as hard analysis of instanton moduli spaces [5]). Thus there have been many conjectures that Donaldson's work may be related to physical ideas in an intimate way. However, such a relation has not been apparent in Donaldson's detailed constructions.

This picture has changed considerably because of the work of Floer on three manifolds [6]. Floer's work involves tunneling amplitudes in $3+1$ dimensions, and has been interpreted by Atiyah [7] in terms of a modified version of supersymmetric quantum gauge theory. (Floer theory has also been reviewed in [8].) In this viewpoint, Floer theory can be seen as a generalization to infinite dimensional function space of the supersymmetric approach to Morse theory [9].

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