Commun. Math. Phys. 117, 243–259 (1988)

Communications in
Mathematical
Physics
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Knots, Links, Braids and Exactly Solvable Models in Statistical Mechanics

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Abstract. We present a general method to construct the sequence of new link polynomials and its two variable extension from exactly solvable models in statistical mechanics. First, we find representations of the braid group from the Boltzmann weights of the exactly solvable models. Second, we give the Markov traces associated with new braid group representations and using them construct new link polynomials. Third, we extend the theory into a two-variable version of the new link polynomials. Throughout the paper, we emphasize the essential roles played by the exactly solvable models and the underlying Yang-Baxter relation.

1. Introduction

In physics we often deal with the configuration problem of one-dimensional objects, for instance, polymers, magnetic fluxes, dislocation lines and trajectories of particles. We generally call a one-dimensional object a string. A knot is a closed string which does not cross with itself. As a more generalized object, an assembly of knots with mutual entanglements is called a link. Classification of knots and links is known to be a longstanding problem in mathematics [1, 2]. In this paper we report an unexpected close connection between physics and mathematics. Namely, we present a general method to construct topological invariants for knots and links by using the theory of exactly solvable models in statistical mechanics.

We begin with the braid and the braid group. Braids are formed when n points on a horizontal line are connected by n strings to n points on another horizontal line directly below the first n points. A trivial n-braid is a configuration where no intersection between the strings is present. A general n-braid is constructed from the trivial n-braid by successive applications of the operation b_i, $i=1,2,\ldots,n-1$. The operation b_i and its inverse b_i^{-1} are best understood by the graphs (Fig. 1). A the set of generators, $b_1, b_2, \ldots, b_{n-1}$, define the braid group B_n [3]. By regarding the trivial n-braid as the identity operation in B_n, we can identify any element in B_n as an n-braid. To guarantee the topological equivalence between different