

Topological Classification of Unknotted Ring Defects

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Abstract. Unknotted ring defects in ordered media are classified in terms of the homotopy theory. It is also investigated what type of point defects will appear when a radius of the ring defect tends to zero.

1. Introduction

The homotopy theory of defects in ordered media has been developed in the field of condensed matter physics (see [1–4] for a review). Topologically stable defects can be classified by means of the homotopy groups of a topological space X which represents internal order of a medium (order parameter space). In the theory, the configurations with defects which can be transformed into each other by continuous deformation are regarded as the same. The topological types of line defects are characterized by conjugacy classes of the first homotopy group (fundamental group), and those of point defects by automorphism classes of the second homotopy group by the action of the first homotopy group. In this paper we develop the mathematical foundation to classify defects of circular shape, or unknotted ring defects which are not penetrated by line defects (Fig. 1). We call the defects of this type the ring defects for short in this paper.

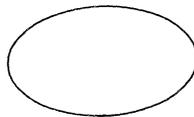


Fig. 1. An unknotted ring defect

In physical situations, ring defects appear when the two line defects which are characterized by mutually inverse elements of the first homotopy group merge. If all the parts of the two line defects approach each other evenly, they disappear at the

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