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# An Exhaustive Criterion for the Non-Existence of Invariant Circles for Area-Preserving Twist Maps

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**Abstract.** We present a necessary and sufficient condition for the non-existence of rotational invariant circles for area-preserving twist maps of the cylinder or annulus based on the "cone-crossing" and "killends" criteria of MacKay and Percival (1985). Given a number of technical restrictions on the implementation of these criteria, this condition leads to a proof of MacKay and Percival's Finite Computation Conjecture.

## 1. Introduction

Much interest has recently been focused on criteria for the non-existence of rotational invariant circles for area-preserving twist maps of the cylinder or annulus [(Chirikov, 1979; Greene, 1979; Mather, 1982, 1984; Newman and Percival, 1983; Aubry, 1983; Katok, 1983; Boyland and Hall, 1985), etc.]. MacKay and Percival (1985) unify a number of related criteria and present a practical algorithm which can be rigorously implemented on a digital computer. Herman (1983) uses essentially the same criterion in his construction of $C^{2-\varepsilon}$ counter-examples to the Moser Twist Theorem whilst Mather (1984) and Aubry (1983) apply this criterion to the standard map:

$$r' = r - (k/2\pi)\sin 2\pi\theta\,,$$

$$\theta' = \theta + r'$$

to show that there are no rotational invariant circles for respectively $|k| > 4/3$ and $|k| > \beta$, where $\beta \simeq 1.23\ldots$ is the root of some transcendental equation. Aubry and Mather's work was done analytically, but by implementing the criterion on a computer MacKay and Percival are able to obtain a considerably better rigorous bound: there are no rotational invariant circles for $|k| \geqq 63/64 = 0.984375$. This is

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