Commun. Math. Phys. 117, 159–176 (1988)

Communications in Mathematical Physics

Moduli of Super Riemann Surfaces

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Abstract. The basic properties of super Riemann surfaces are presented, and their supermoduli spaces are constructed, in a manner suitable for the application of algebro-geometric techniques to string theory.

0. Introduction

Super Riemann surfaces were introduced by Friedan [Fr] as the correct supersymmetric analogue of a Riemann surface, and the supermoduli space plays a role in superstring theory analogous to the role of moduli space in bosonic string theory. In this paper we will provide a description of supermoduli space in precise sheaf-theoretic terms, following the classical lines of Kodaira-Spencer deformation theory.

We will in fact construct the supermoduli spaces for super Riemann surfaces with level-n structure, $n \geqq 3$. The choice of $n=3$ as the lower limit is dictated by the fact that 3 is the least of all integers n such that an automorphism of a Riemann surface $(\Sigma, \mathcal{O})$ inducing the identity on $H^1(\Sigma, \mathbf{Z}/n\mathbf{Z})$ is itself the identity map [F–K]. It follows that the *reduced* space of the level-n supermoduli space is nonsingular. Nevertheless, the supermoduli spaces are *orbifolds*, so to speak, in the "odd" directions. Each is locally the quotient of a supermanifold by the canonical automorphism which sends any superfunction f of definite parity $|f|$ to $(-1)^{|f|}f$. The construction calls attention to the topological problem of determining, for each value of n, whether the level-n supermoduli space is a *global* orbifold. This is equivalent to asking whether the spin structures on the fibers of the universal curve over the reduced space may be fitted together to form a square root of the canonical bundle of that curve. Some remarks about this problem are given in the appendix.

One may also consider super Riemann surfaces with a homotopy marking. The corresponding supermoduli space *is* a global orbifold. It is the quotient, by the

* Supported in part by NSF Grant No. DMS-8704401
** Supported in part by NSF Grants No. DMS-8501783 and No. DMS-86107301(1)