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Indefinite Harmonic Forms and Gauge Theory

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Abstract. Indecomposable representations have been extensively used in the construction of conformal and de Sitter gauge theories. It is thus noteworthy that certain unitary highest weight representations have been given a geometric realization as the unitary quotient of an indecomposable representation using indefinite harmonic forms [RSW]. We apply this construction to $SU(2,2)$ and the de Sitter group. The relation is established between these representations and the massless, positive energy representations of $SU(2,2)$ obtained in the physics literature. We investigate the extent to which this construction allows twistors to be viewed as a gauge theory of $SU(2,2)$. For the de Sitter group, on which the gauge theory of singletons is based, we find that this construction is not directly applicable.

I. Introduction

Representations of space-time symmetry groups play a fundamental role in physics. In particular, minimal energy, indecomposable representations [A], in which the physical sector is realized on a unitary subquotient, have been extensively used in the construction of Poincaré, de Sitter, and conformal gauge theories ([AFFS, BFH1, 2, F, FF1-6, FPS, Ha], and references therein). Especially interesting is the recent progress in singletons ([FF1-6]), a theory which has its origins in the work of Dirac [D2]. It is thus extremely interesting that in the recent mathematical literature, certain representations of this type have been given a very elegant geometric realization in terms of indefinite harmonic forms [RSW]. Several physical applications of this indefinite harmonic construction are immediately suggested, and these possibilities will be investigated in this note. First, let us introduce the representation in question.

Recall that the unitary irreducible representations of semisimple Lie groups can be divided into two classes: the regular representations, which occur in the Plancherel decomposition of $L_2(G)$, and the singular representations, which do not. More precisely, the representations treated in [RSW] are highest weight representations lying in the analytic continuation of the holomorphic discrete series (which we take to include those discrete points known as the "Wallach set"). Basically, the construction is the following. Given a Lie group G, a subgroup H