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Fermionic Fields on $\mathbb{Z}_N$-Curves

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Abstract. The line bundles of degree $g-1$ on $\mathbb{Z}_N$-curves corresponding to $1/N$ nonsingular characteristics are considered. The determinants of Dirac operators defined on these line bundles are evaluated in terms of branch points. The generalization of Thomae's formula for $\mathbb{Z}_N$-curves is derived.

1. Introduction

In this paper we continue our investigations of conformal field theories that are induced in some specific way from algebraic curves. Our global strategy will be the following. Consider an algebraic curve represented as an N-sheeted ramified covering over CP^1. Such representation fixes the unique singular metric whose projection onto CP^1 is $dzd\bar{z}$. The determinants of different operators defined for line bundles of definite degree and corresponding to some characteristics can be represented as correlation functions of some conformal fields defined on a complex plane. We call such conformal fields twist operators or σ-fields. These operators simulate the proper monodromy behaviour and make the fields be multivalued fields defined not on a complex plane but on a ramified covering. This ideology takes its origin in the works of Sato et al. [1]. Our investigations of this problem were highly stimulated by a recent work by Zamolodchikov [2], who succeeded in representing the determinant of the scalar Laplacian defined on a hyperelliptic curve as a correlation function of some spin operators from the Ashkin-Teller model.

It turns out that the twist operators are actually the conformal fields with respect to the full stress-energy tensor

$$T(z)=\sum_{\text{sheets}} t^n(z). \tag{1.1}$$

$T(z)$ is a single-valued function on CP^1 that may only have poles at the branch points. To evaluate the conformal dimensions one has to analyze the behaviour of $T(z)$ in vicinities of the branch points. In a vicinity of the branch point the variable z defined on a complex plane is not a proper coordinate on the covering. The proper