Commun. Math. Phys. 116, 659–674 (1988)

Communications in Mathematical Physics

Non-Linear Multi-Plane Wave Solutions of Self-Dual Yang-Mills Theory

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Abstract. New solutions of self-dual Yang-Mills (SDYM) equations are constructed in Minkowski space-time for the gauge group $SL(2, \mathbb{C})$. After proposing a Lorentz covariant formulation of Yang's equations, a set of Ansätze for exact non-linear multiplane wave solutions are proposed. The gauge fields are rational functions of $e^{x\cdot k_i}(k_i^2=0, 1\leqq i\leqq N)$ for these Ansätze. At least, three families of multisoliton type solutions are derived explicitly. Their asymptotic behaviour shows that non-linear waves scatter non-trivially in Minkowski SDYM.

1. Introduction

Integrable theories in $1+1$ dimensions have been developed very successfully in the last years. It is then a natural problem to investigate their multidimensional analogues. Namely four- (or n-) dimensional field theories having an associate linear differential system with a spectral parameter(s). Such linear systems are known for self-dual (and antiself-dual) Yang-Mills equations (SDYM) [1, 2] and SUSY Yang-Mills [3], as well as for non-Lorentz invariant equations like Kadomtsev-Petviashvili (KP), or three-wave equations in $2+1$ dimensions [4, 6]. Actually, the dynamics of KP is known much better than that of SDYM in Minkowski space-time.

The construction of multi-soliton (non-linear multi-plane wave) solutions of SDYM in $3+1$ dimensions is the purpose of this paper. The SDYM reads there

$$F_{\mu\nu}=\frac{i}{2}\,\varepsilon_{\mu\nu\lambda\sigma}F^{\lambda\sigma} \tag{1.1}$$

(here $\varepsilon_{0123}=+1$). Since an explicit factor (i) appears, these equations describe complex solutions for $SU(N)$ gauge fields or equivalently real solutions (real gauge

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