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A Convergence Theorem for Lattice Feynman Integrals with Massless Propagators

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Abstract. It is shown that for non-vanishing lattice spacing, conventional infrared power counting conditions are sufficient for convergence of lattice Feynman integrals with zero-mass propagators. If these conditions are supplemented by ultraviolet convergence conditions, the continuum limit of such a diagram exists and is universal.

1. Introduction

In a recent paper [1] we have proposed a convergence theorem, which states existence of the continuum limit for a wide class of Feynman integrals with a lattice cutoff if certain ultraviolet (UV) power counting conditions are satisfied. What is counted are lattice divergence degrees in Zimmermann subspaces, i.e. in affine subspaces of the integration momenta. To avoid infrared (IR) singularities, we had assumed all propagators to be massive. In the present article we extend the considerations to integrals containing zero-mass propagators. While the lattice provides a UV-cutoff, IR-singularities are expected to be quite the same as for continuum diagrams. As will be shown, IR-power counting conditions similar as for continuum diagrams [2–5] are sufficient to guarantee the convergence of lattice Feynman integrals, at least for non-vanishing lattice spacing. If these conditions are supplemented by the UV-power counting conditions of [1], the continuum limit of the Feynman integral exists and coincides with the formal limit, i.e. it is given by the integral resulting from the $a \to 0$-limit in the integrand.

This article is organized as follows. At first, in Sect. 2, the notion of an IR-degree is introduced in a form which is similar to the definition of a UV-degree in [1]. The power counting theorem for Feynman integrals with zero-mass propagators is formulated in Sect. 3. As in the massive case, the denominator of a Feynman integrand can easily be treated, whereas the numerator must be estimated in such a way that UV- as well as IR-power counting conditions are taken into account

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