Commun. Math. Phys. 116, 539–572 (1988)

Communications in Mathematical Physics

Phase Transition in the 3d Random Field Ising Model

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Abstract. We show that the three-dimensional Ising model coupled to a small random magnetic field is ordered at low temperatures. This means that the lower critical dimension, d_l for the theory is $d_l \leqq 2$, settling a long controversy on the subject. Our proof is based on an exact Renormalization Group (RG) analysis of the system. This analysis is carried out in the domain wall representation of the system and it is inspired by the scaling arguments of Imry and Ma. The RG acts in the space of Ising models and in the space of random field distributions, driving the former to zero temperature and the latter to zero variance.

1. Introduction

An interesting class of disordered systems is obtained by coupling impurities to the order parameter of a statistical system. This situation may be modelled e.g., by the Ising model (or the ϕ^4-theory) with a random magnetic field. This random field Ising model (RFIM) describes actual physical systems, such as dilute antiferromagnets in a uniform field [1] and has been used to study, among other things, the effects of impurities on the fluctuations of interfaces [2, 3]. It has also served as a useful playground for various theoretical ideas: the replica trick, [4, 5] dimensional reduction [6, 7, 8, 9] and supersymmetry [10].

As usual, one of the interesting theoretical questions is to determine the upper and lower critical dimensions of the model. The most elegant argument for the upper critical dimension d_u (i.e., the dimension above which the theory is Gaussian in the infrared) is dimensional reduction, which says that, at long distances and near d_u, the (quenched) correlation functions for the random system behave as those of the corresponding deterministic system, but in two less dimensions. The argument was, basically, to replace the random system by its tree approximation, a stochastic differential (or difference) equation, which, *via* its representation in

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