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Periodic Solutions of Some Infinite-Dimensional Hamiltonian Systems Associated with Non-Linear Partial Difference Equations I

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Abstract. We establish existence of a dense set of non-linear eigenvalues, E, and exponentially localized eigenfunctions, u_E, for some non-linear Schrödinger equations of the form

$$Eu_E(x) = [(-\Delta + V(x))u_E](x) + \lambda u_E(x)^3,$$

bifurcating off solutions of the linear equation with $\lambda = 0$. The points x range over a lattice, $\mathbb{Z}^d$, $d = 1, 2, 3, \ldots$, Δ is the finite difference Laplacian, and $V(x)$ is a random potential. Such equations arise in localization theory and plasma physics. Our analysis is complicated by the circumstance that the linear operator $-\Delta + V(x)$ has dense point spectrum near the edges of its spectrum which leads to small divisor problems. We solve these problems by developing some novel bifurcation techniques. Our methods extend to non-linear wave equations with random coefficients.

0. Introduction. Motivation, Results, and Basic Ideas

The purpose of this paper is to construct infinitely many time-periodic solutions to some non-linear, partial difference equations which can be viewed as the equations of motion of Hamiltonian systems with infinitely many degrees of freedom. Physically, these systems describe infinite arrays of coupled anharmonic oscillators with the property that when the anharmonic (non-quadratic) terms in the Hamilton function are neglected the frequencies of the oscillators are non-resonant, in a suitably strong sense to be made precise later on. We propose to show that from infinitely many periodic solutions of the unperturbed system of harmonic oscillators periodic solutions of the perturbed system of coupled anharmonic oscillators bifurcate.

The main difficulty encountered in such an attempt is that the spectrum of frequencies of the unperturbed system is dense in some interval $I \subseteqq \mathbb{R}$. This makes standard bifurcation techniques inapplicable and has motivated us to develop

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